Starter Question

The straight line L passes through the points $(2,5)$ and $(-2,3)$, and meets the coordinate axes at the points P and Q.

Find the area of a square whose side is PQ.

GRADIENT OF L

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{-2-2} = \frac{-2}{-4} = \frac{1}{2}$$

$P(0,4)$

$Q(-8,0)$

EQUATION OF L, GRADIENT $\frac{1}{2}$, PASSING THROUGH $(2,5)$

$\Rightarrow y - y_0 = m(x - x_0)$

$\Rightarrow y - 5 = \frac{1}{2}(x-2)$

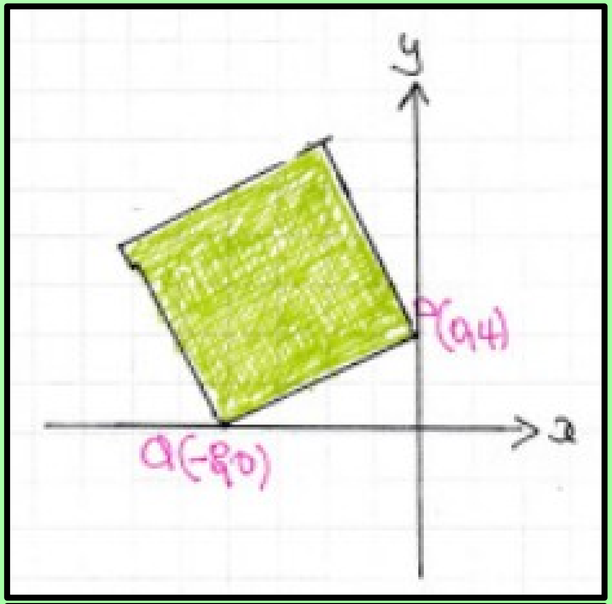
$\Rightarrow 2y - 10 = x - 2$

$\Rightarrow 2y = x + 8$

$|PQ| = (0-4)^2 + (-8-0)^2$

$|PQ| = \sqrt{16+64}$

$|PQ| = \sqrt{80}$



$\sqrt{80} \times \sqrt{80} = 80$

Hand in your baseline intervention homework

C Coordinate geometry in the (x, y) plane

C2

Understand and use the coordinate geometry of the circle including using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$; completing the square to find the centre and radius of a circle; use the following properties:

- the angle in a semicircle is a right angle
- the perpendicular from the centre to a chord bisects the chord
- the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point.

C Coordinate geometry in the (x, y) plane

Assessed at AS and A-level

Teaching guidance

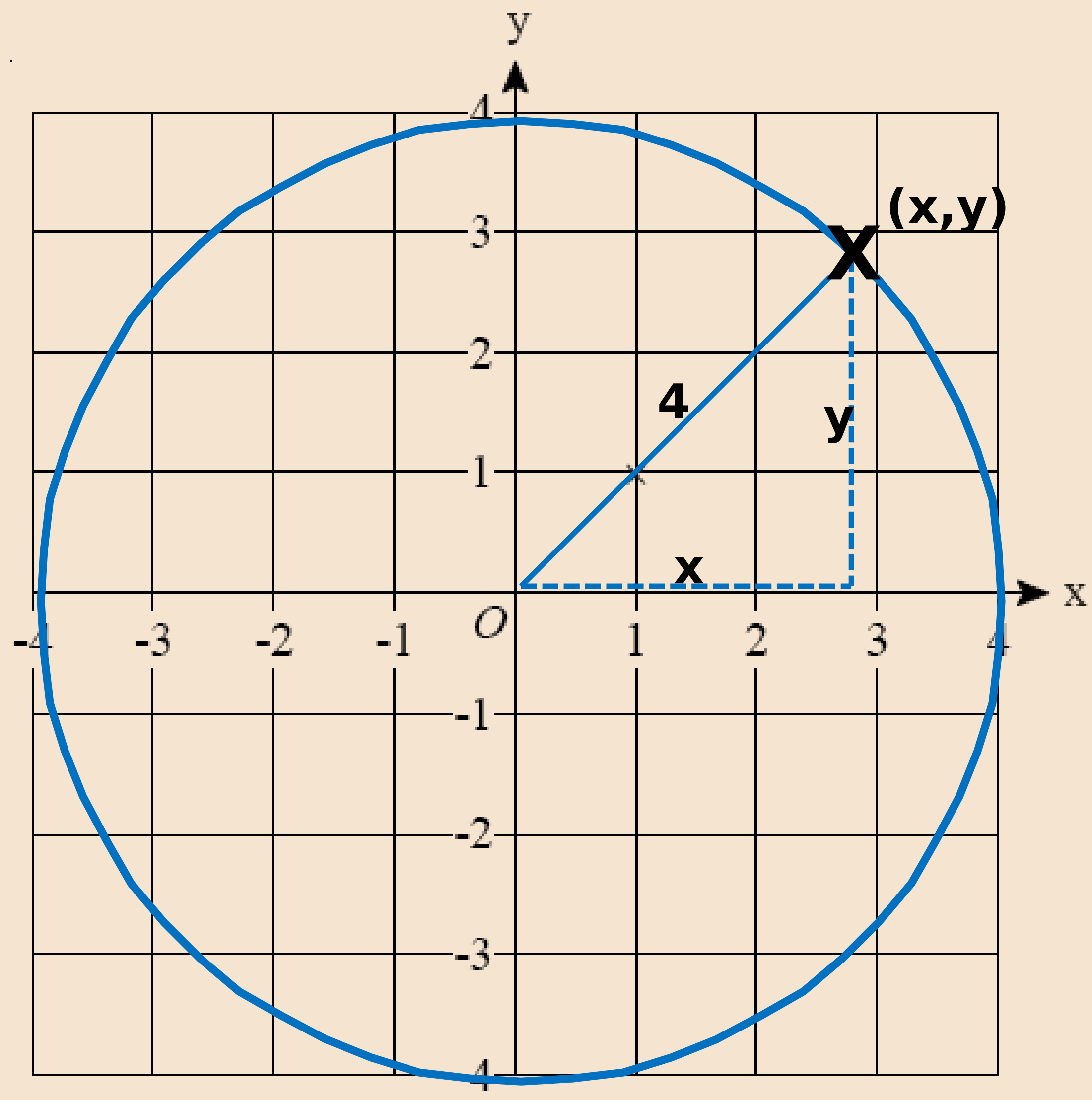
Students should be able to:

- find the equation of a tangent or normal at a point
- find relevant gradients using the coordinates of appropriate points.

Note: implicit differentiation of the equation of a circle will not be required at AS, but could be expected at A-level.

1.6 Circle Geometry

Tuesday, May 27, 2025



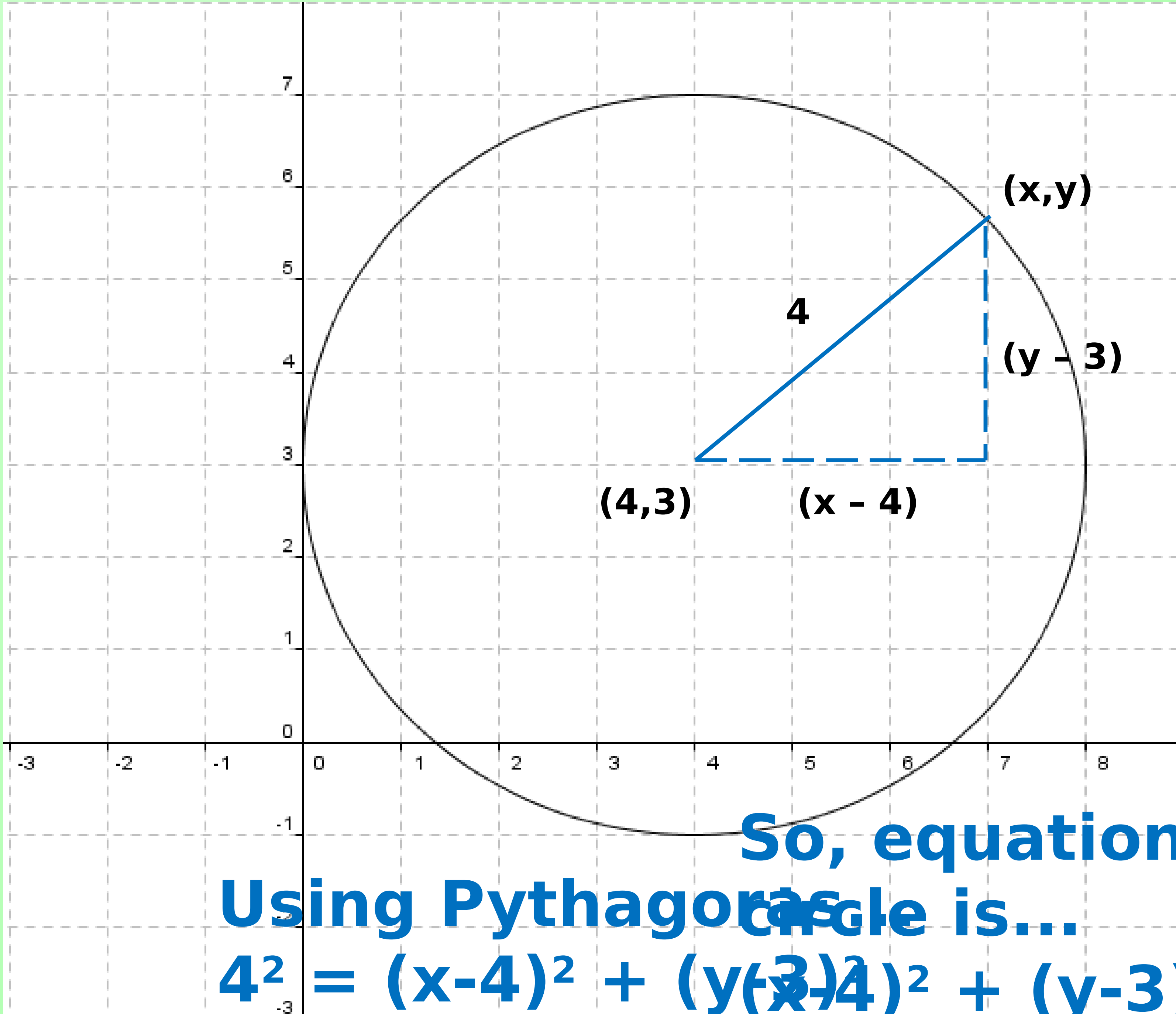
Find the equation of a circle, centre (0,0) and radius 4.

Using Pythagoras

$4^2 = x^2 + y^2$

So, equation of circle is...

$x^2 + y^2 = 16$



Using Pythagoras…

$4^2 = (x-4)^2 + (y-3)^2$

So, equation of circle is…

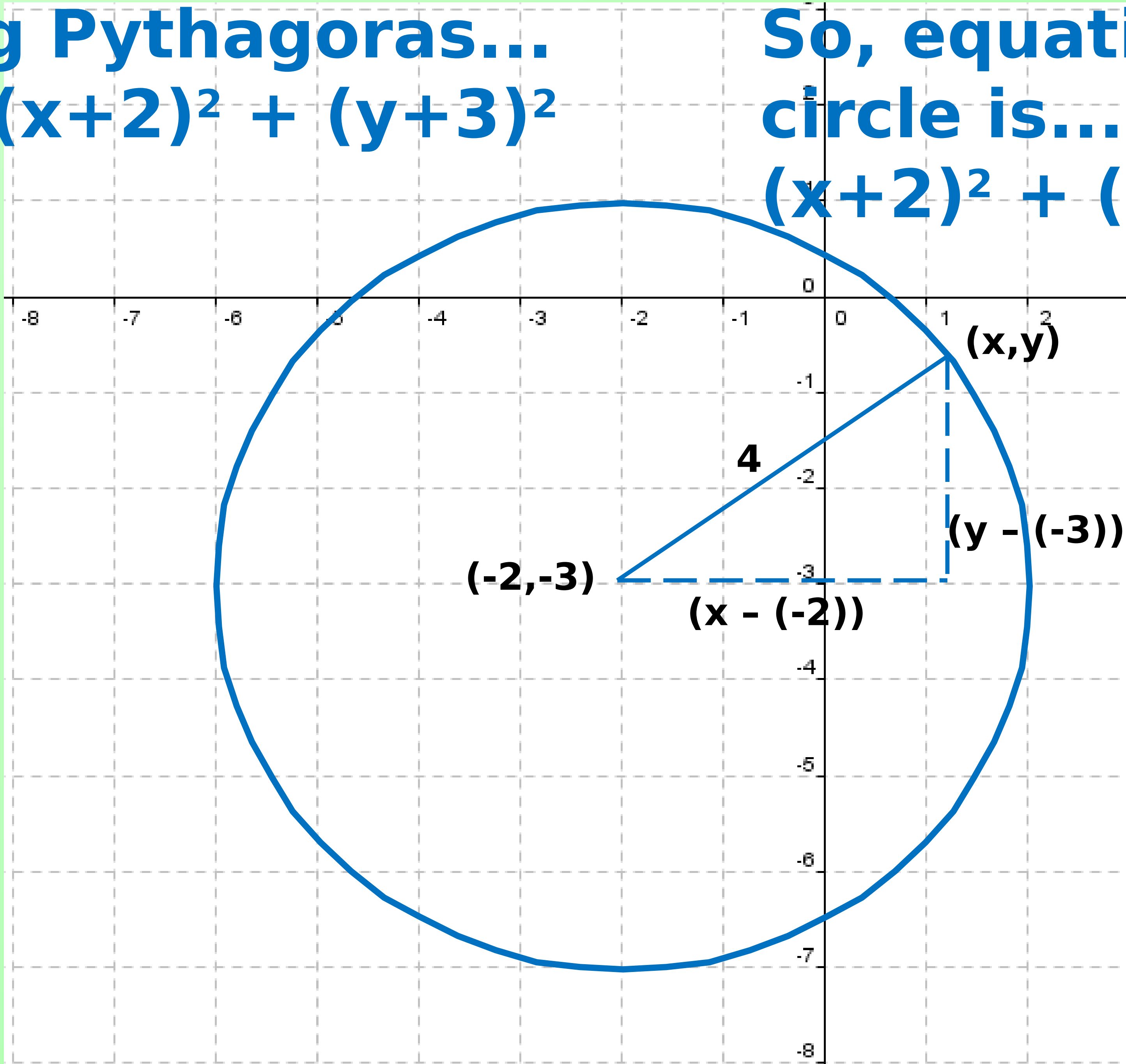
$(x-4)^2 + (y-3)^2 =$

sing Pythagoras…

$= (x+2)^2 + (y+3)^2$

So, equation of circle is…

$(x+2)^2 + (y+3)^2 =$



1.6: Circle Geometry

Tuesday, May 27, 2025

The general equation of a circle centred on (0,0) with radius r is given by:

The general equation of a circle centred on (a, b) with radius r is given by:

This can be expanded and simplified:

1.6: Circle Geometry

Tuesday, May 27, 2025

Example 1

Work out the equation of the circle with centre $(-4, 9)$, radius $\sqrt{8}$

Write your answer without brackets.

1.6: Circle Geometry

Example 2

The equation of a circle is $x^2 + y^2 - 6x + 4y + 4 = 0$. Find the centre of the circle and the radius.

Centre: (3, -2)
Radius: 3

You try: $x^2 + y^2 + 4x - 8y - 44 = 0$

The Centre is $(-2, 4)$ and radius $= 8$.

1.6: Circle Geometry

Example 3

The line segment AB is a diameter of a circle, where A and B are (4, 7) and (−8, 3) respectively. Find the equation of the circle.

To find the radius, find the length of AB and divide by two…

To find the centre, find the mid-point of AB…

1.6: Circle Geometry

Example 4

Does the point (5,15) lie on the circle with centre (5,5) and a radius of 10?

Yes the point lies on the circle.

1.6: Circle Geometry

Do questions 1-8

Example 5

Does the point (1,3) lie on the circle $(x-2)^2+(y-1)^2=25$?

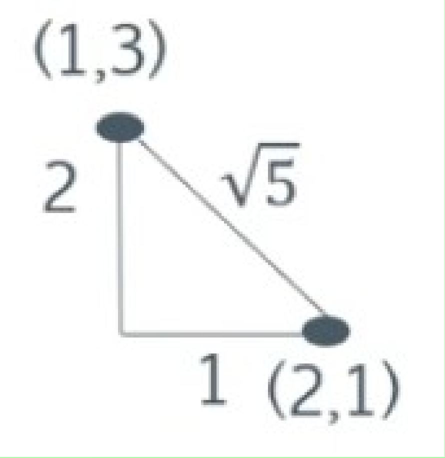
$$(x-2)^2+(y-1)^2=25$$

$$(1-2)^2+(3-1)^2=25$$

$$(-1)^2+(2)^2=25$$

$$5=25$$

$$So\ not\ on\ the\ circle$$



$$distance=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$distance=\sqrt{(1-2)^2+(3-1)^2}$$

$$distance=\sqrt{5}$$

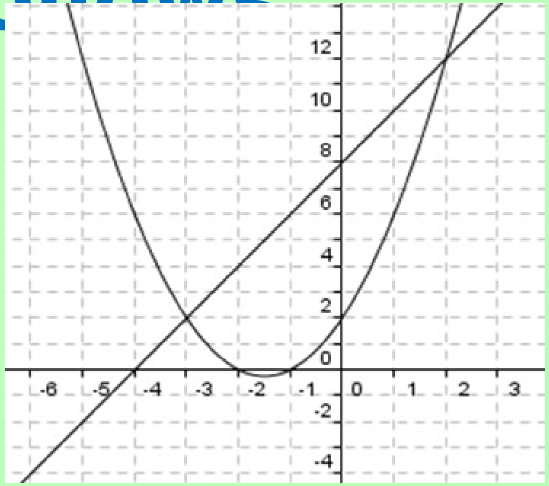
Is it inside or outside?

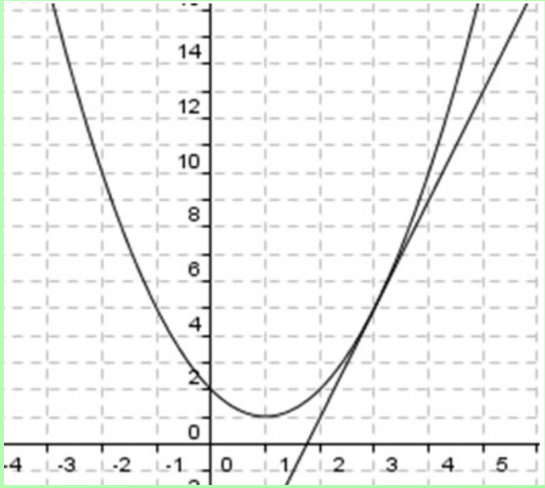
$$\underline{\sqrt{5}<5\ so\ inside\ circle}$$

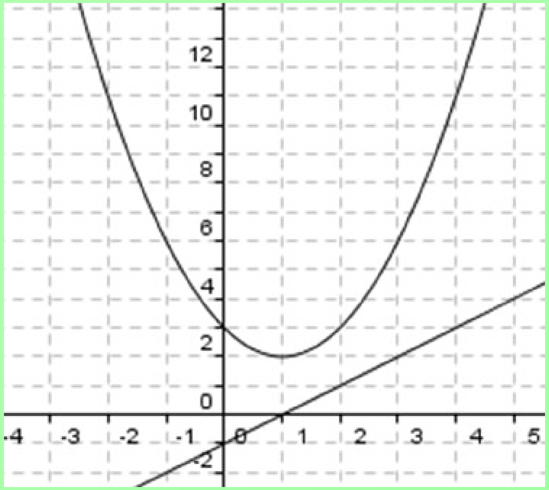
Tuesday, May 27, 2025

1.6 Circle Geometry

We saw previously that straight lines and quadratics can intersect as follows:





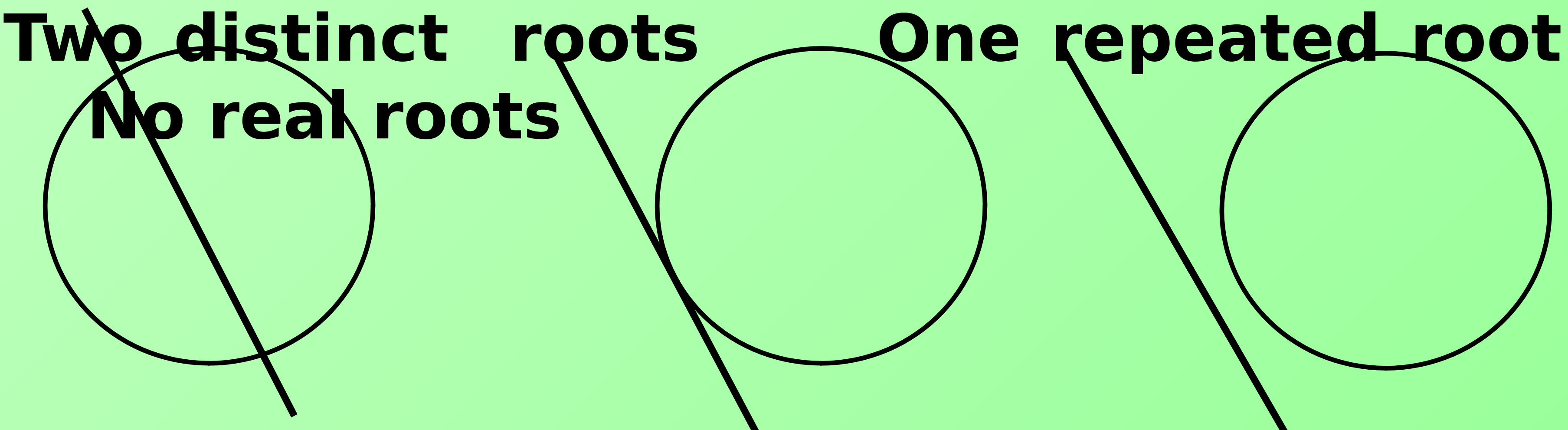


Two distinct roots **One repeated root**

No real roots

1.6 Circle Geometry

The same three possibilities apply to the **intersection** of a straight line and a circle. To decide how many points of intersection there are you must **solve simultaneously** the equations of the straight line and the circle. The number of roots of the resulting quadratic determines which case applies:



1.6 Circle Geometry

Example 6:

Show that the line $y = x - 7$ does not meet the circle $(x + 2)^2 + y^2 = 33$.

Hence there are no real roots and therefore no points of intersection between the line and the circle.

1.6 Circle Geometry

Example 7:

A circle has equation $x^2 + y^2 - 4x - 14 = 0$.

A line has equation $y = 2k - x$, where k is a constant.

(i) Show that the x-coordinate of any point of intersection of the line and the circle satisfies the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$

(3 marks)

1.6 Circle Geometry

Example 7:

A circle has equation $x^2 + y^2 - 4x - 14 = 0$.

A line has equation $y = 2k - x$, where k is a constant.

(ii) Find the values of k for which the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$

has equal roots. *(4 marks)*

Equal roots when

1.6 Circle Geometry

Do questions 9-15

Example 7:

A circle has equation $x^2 + y^2 - 4x - 14 = 0$.

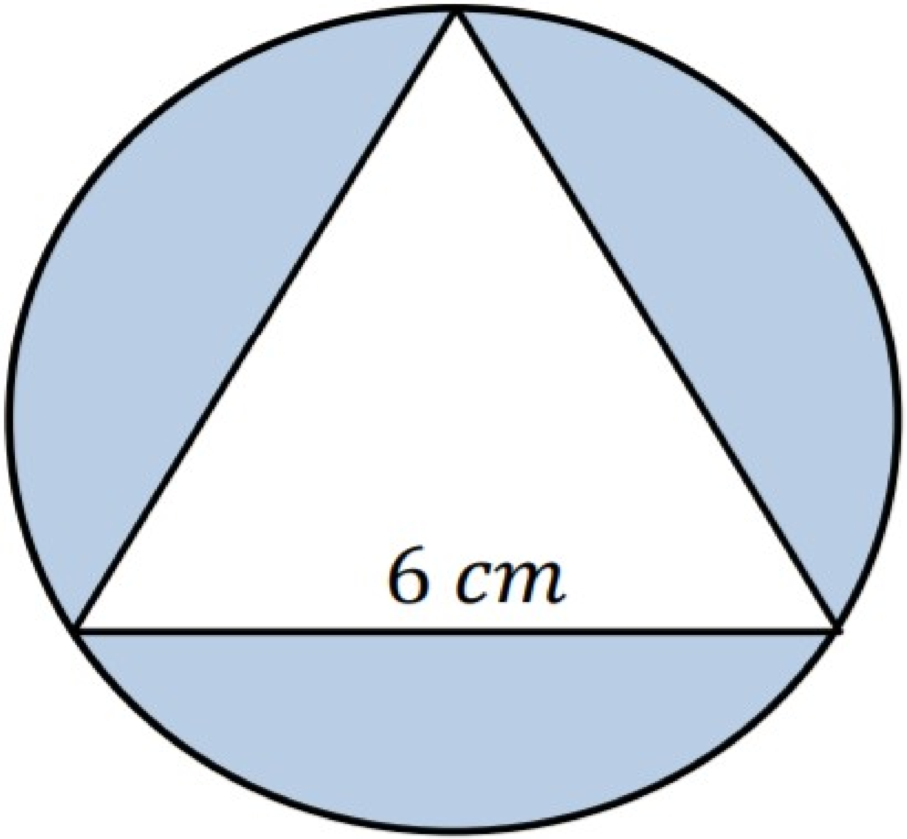
A line has equation $y = 2k - x$, where k is a constant.

(iii) Describe the geometrical relationship between the line and the circle when k takes either of the values found in part (c)(ii). *(1 mark)*

Equal roots only one point of intersection so the line would be a tangent to the circle.

1.6 Circle Geometry

This diagram shows an equilateral triangle of side length 6 cm drawn inside a circle so that each corner touches the circumference of the circle.



What area of the circle is shaded?